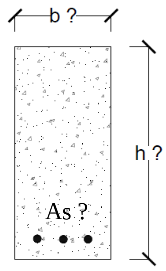
2. Design of rectangular beam with tension reinforcement (singly reinforcement) with non-specified dimensions

- In this design problem, there are no previous functional on beam dimensions
- Then the main unknowns in the design process are three parameters:
  i. **Beam width "b".**
  ii. **beam depth "h".**
  iii. **Reinforcement ratio ρ**



Design Procedure

1. Compute required factored applied moment **Mu** based on given load (dead and live loads).
2. As we have one relation, namely:

   $Mu=\varphi Mn=\varphi \rho f_y bd^2(1-0.59\frac{\rho f y}{fc`})$ **assume** φ=0.9 and will be check later.

   Since we have three unknowns **(b,d and ρ)**. Then **two assumptions** to be adopted in this design procedure:

   i. **Select a reinforcement ratio (first assumption):**
      - Theoretically, reinforcement ratio can be selected anywhere between maximum and minimum steel ratios **(ρ_{min} to ρ_{max}).**
      - For economical design will have reinforcement ration between **0.5** ρ_{max} *to* $0.75\ \rho_{max}$.
      - Usually this ratio will be mentioned in a question.

   ii. **Assume b/d ratio (second assumption)**

      Experience and judgment developed over years have also established a range of acceptable and economical depth/width ratio for rectangular beam:

      $1.5 \leq \frac{d}{b} \leq 3$

      - Usually this ratio will be mentioned in a question.

3. Substitute both of selected $\boldsymbol{\rho}$ and ratio of ($\frac{d}{b}$)in the main equation:

   $$Mu=\varphi \mathbf{Mn}=\varphi\, \boldsymbol{\rho f_y} \mathbf{bd^2(1\text{-}0.59}\frac{\rho fy}{fc`})$$

   And get d and b

   - Round both of **b** and **d** to nearest **25 mm**
4. Compute required steel area

   As $_{required}$ = ρ*(bd)
5. Compute the required number of bars (n)

   No. of bars (n) = $\frac{As}{Abar}$

   - Round up the number to nearest integer.
6. Check if rebars can in be put in single layer

   $b_{required}$ = 2 × cover +2 ×stirrups diameter + No. of rebars × bar diameter + (No. of rebars-1) × spacing between rebars

   If

   $b_{required} > b_{aviable}$

   Then reinforcement cannot be put in a single layer.

   S_{clear} = larger [25 mm, d_b]
7. Compute the depth (h)

   h $_{\text{for one layer}}$= d + cover + stirrups + $\frac{\textbf{bar diameter}}{\textbf{2}}$

   h $_{\text{for two layer}}$= d + cover + stirrups + bar diameter + $\frac{\textbf{spacing between layers}}{\textbf{2}}$

   - Round the computed "h" to nearest 25mm
8. Check the assumption of φ=0.9

   $a=\frac{As*fy}{0.85fc`*b}$

   $c=\frac{a}{\beta 1}$

   $\epsilon_t=\frac{dt-c}{c}\epsilon_u$ where: ϵ_u=0.003

- If $\epsilon t \geq 0.005$, then φ=0.9
- If $\epsilon t < 0.005$ then

  $\varphi=0.483+83.3\times\epsilon_t$

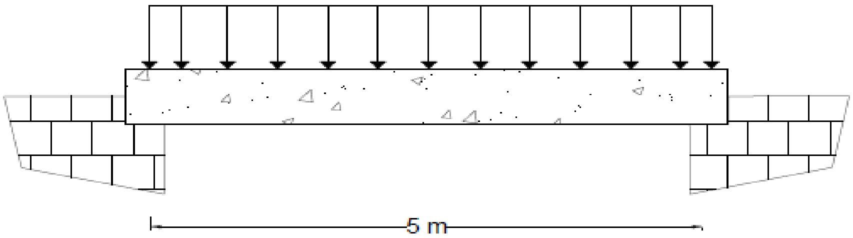
  And retain to step 3

9. Draw final detailed section

Example1: Design a simply supported rectangular reinforced concrete beam shown in Figure below.

Assume that the designer intends to use:

- Concrete of $fc` = 25$ Mpa.
- Steel $fy = 420$ Mpa.
- Use $\rho = 0.5\rho max$ and $\frac{d}{b} = 2$
- Rebar diameter 25mm for longitudinal reinforcement.
- Rebar diameter 10mm for stirrups.
- W_D=15 kN/m**(including self-weight)**= and W_L=20 kN/m



Solution:

1. Compute required factored applied moment **Mu**

W_D=15 kN/m

W_L=20 kN/m

$W_U = 1.2W_D + 1.6W_L$

$W_U = 1.2*15 + 1.6*20$

W_U=50 kN/m

$M_u = \frac{Wu\ell^2}{8} = \frac{50*5^2}{8} = 156.25$ kN.m

2.

$$Mu = \varphi \boldsymbol{Mn} = \varphi\, \boldsymbol{\rho f_y bd^2(1\text{-}0.59\frac{\rho fy}{fc`})}$$

As it was mentioned in question $\rho = 0.5\rho_{max}$ and $\frac{d}{b} = 2$

$\rho_{max} = 0.85*\beta_1 \frac{fc`}{fy} \frac{\epsilon u}{\epsilon u + 0.004}$ and $\epsilon_u = 0.003$

$\rho_{max} = 0.85*0.85*\frac{25}{420} \frac{0.003}{0.003+0.004} = 0.0184$

$\rho = 0.5\rho_{max} = 0.5 \times 0.0184 = 9.2 \times 10^{-3}$

3. Substitute both of selected ρ and ration of $\frac{d}{b}$ in the main equation:

$$Mu=\varphi \mathbf{Mn}=\varphi\, \rho f_y \mathbf{bd^2(1\text{-}0.59}\frac{\rho fy}{fc`})$$

$$156.25\times10^6 = 0.9\times9.2\times10^{-3}\times420\times bd^2\ (1-0.59\times\frac{9.2\times10^{-3}*420}{25})$$

$$156.25\times10^6 = 3.16\ \times bd^2$$

$$bd^2 = 49.44\ \times10^6\ mm^2$$

As $\frac{d}{b}=2$ ► d=2b

$$(b\times(2b)^2) = 49.44\ \times10^6\ mm^2$$

$$4b^3 = 49.44\ \times10^6\ mm^2$$

$$b^3 = 12.36\ \times10^6\ mm^2$$

$$b = \sqrt[3]{12.36\ \times10^6}$$

b =231.2 mm ≈250 mm

Then “d” will be:

$\frac{d}{250}=2$ ►500 mm

4. Compute required steel area

$As_{\text{required}} = \rho*(bd) = 9.2\times10^{-3}\times250\times500 = 1150\ mm^2$

5. Compute the required number of bars (n)

No. of bars (n) = $\frac{\boldsymbol{As}}{\boldsymbol{Abar}} = \frac{1150}{\frac{\pi}{4}25^2} = \frac{1150}{491} = 2.34 \approx 3$

6. Check if rebars can in be put in single layer

b_{required} = 2 × cover +2 × stirrups diameter + No. of rebars × bar diameter + (No. of rebars-1) × spacing between rebars

b_{required} = 2 *40+2*10+3*25+2*25 = 225 mm

$b_{\text{required}} = 225 < b_{\text{provided}} = 250$ O.K

7. Compute the depth (h)

$h_{\text{for one layer}} = d + cover + stirrups + \frac{bar\ diameter}{2}$

$h_{\text{for one layer}} = 500 + 40 + 10 + \frac{25}{2} = 562.5 \approx 565$ mm

Use (250 × 565) mm with 3φ25 mm

8. Check the assumption of φ=0.9

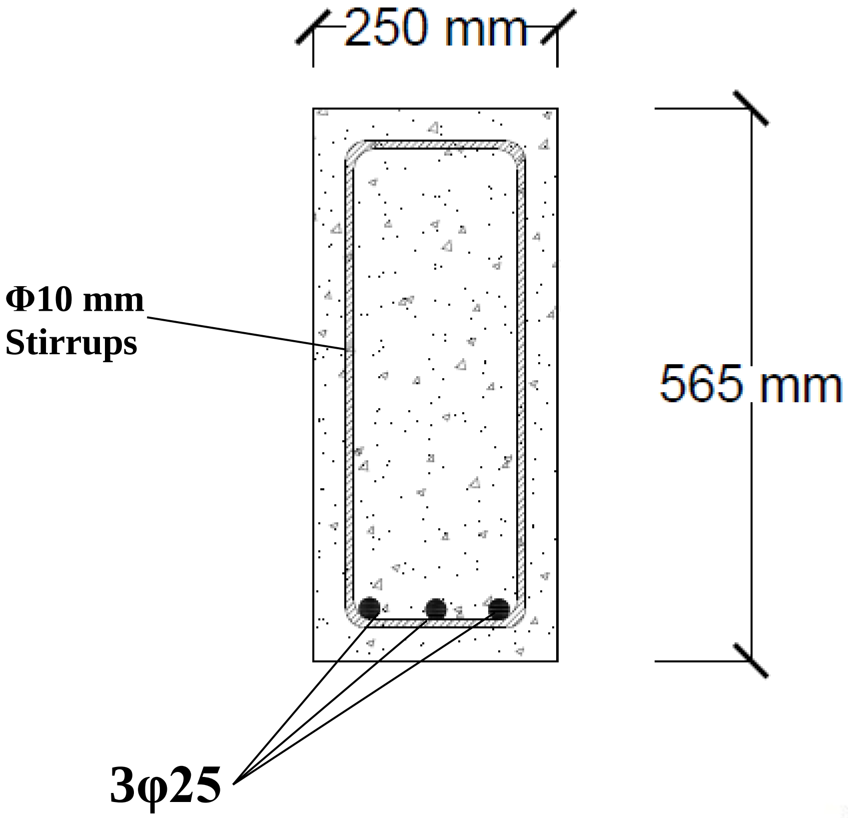
$$a = \frac{As*fy}{0.85fc`*b} = \frac{1150*420}{0.85*25*250} = 90.9\ mm$$

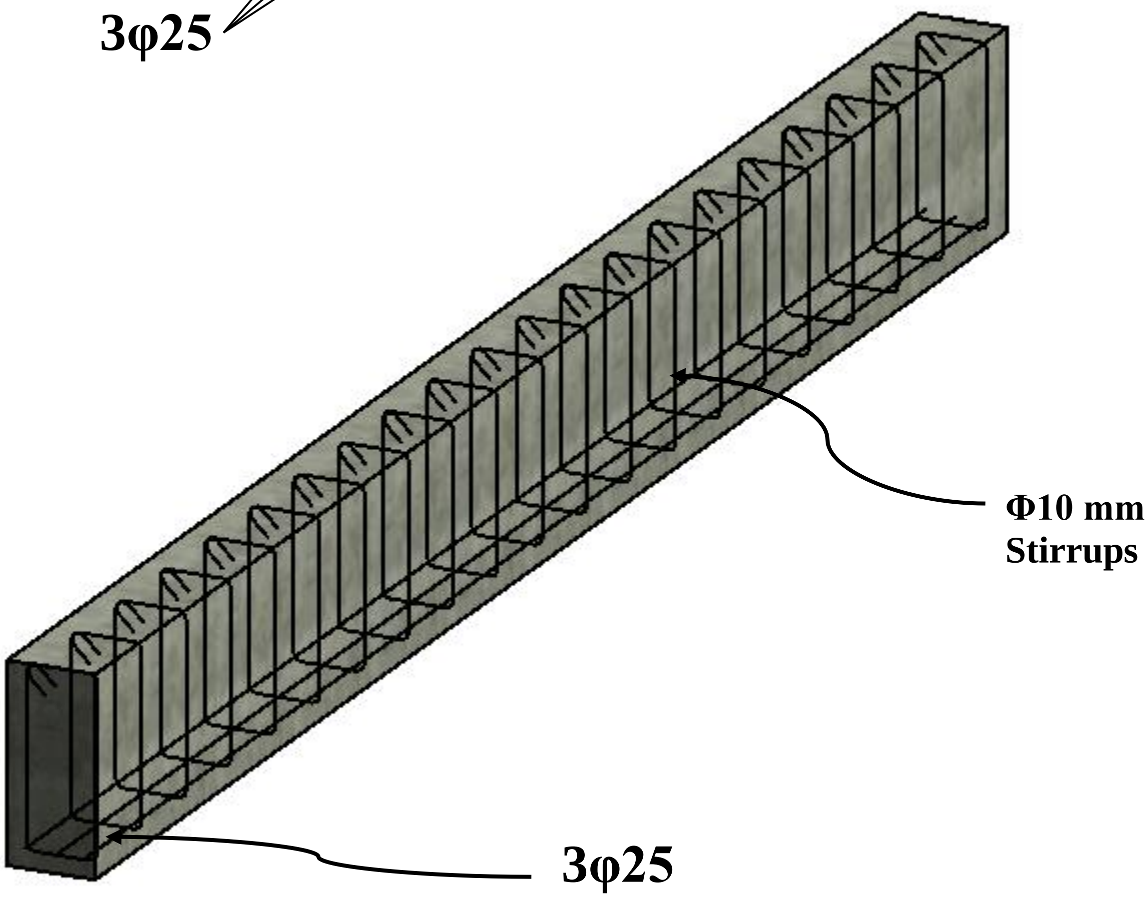
$$c = \frac{a}{\beta1} = \frac{90.9}{0.85} = 106.9\ mm$$

$$\epsilon_t = \frac{dt-c}{c}\epsilon u = \frac{500-106.9}{106.9} * 0.003 = 0.011 > 0.005$$

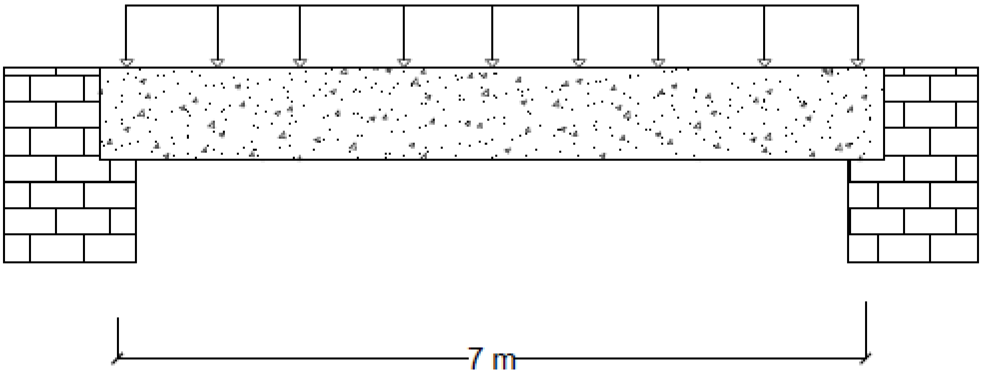
$\therefore \varphi = 0.9$

9. Draw final detailed section





Example 2 : Design beam show in figure below for flexure requirements according to ACI code



Assume the designer intends to use:

- $\rho=0.5\rho_{max}$
- h=550 mm .
- fc`=25 Mpa, fy=420 Mpa .
- Rebar 16mm for longitudinal reinforcement.
- Rebar 10mm for stirrups.
- Two layers of reinforcements.

Solution:

1. Compute required factored applied moment **Mu**

W_D=10 kN/m

W_L=15 kN/m

$W_U=1.2W_D+1.6W_L$

W_U=1.2*10+1.6*15

W_U=36 kN/m

$M_u=\frac{Wu\ell^2}{8}=\frac{36*7^2}{8}$ =220.5 kN.m

2. $Mu=\varphi \mathbf{Mn}=\varphi\, \rho f_y \mathbf{bd^2(1\text{-}0.59}\frac{\rho fy}{fc`})$

As it was mentioned in question $\rho=0.5\rho_{max}$ and h=600 mm

$\rho_{max}=0.85*\beta_1\frac{fc`}{fy}\frac{\epsilon u}{\epsilon u+0.004}$ and ϵ_u =0.003

$\rho_{max}=0.85*0.85*\frac{25}{420}\frac{0.003}{0.003+0.004}$ =0.0184

$\rho=0.5\rho_{max}=0.5\times0.0184=9.2\times10^{-3}$

3. Substitute both of selected $\boldsymbol{\rho}$ and ration of $\frac{d}{b}$ in the main equation:

   $Mu=\varphi \mathbf{Mn}=\varphi\, \rho f_y \mathbf{bd^2(1\text{-}0.59}\frac{\rho f y}{fc`})$

   $\mathbf{d}_{\text{for two layer}}$ **= 550 –cover-stirrups-bar diameter+**$\frac{\textit{spacing between layers}}{2}$

   $d_{\text{for two layer}} = 550 - 40 - 10 - 16 - \frac{25}{2} = 471.5$ mm

   $220.5*10^6 = 0.9*9.2\times10^{-3}*420*b*471.5^2*(1-0.59*\frac{9.2\times10^{-3}*420}{25})$

   $220.5*10^6 = 702612.58*b$

   $b = 313.8 \approx 325$ mm

4. Compute required steel area

   $As_{\text{required}} = \rho*(bd) = 9.2\times10^{-3}\times325\times471.5 = 1409.8\ mm^2$

5. Compute the required number of bars (n)

   No. of bars (n) = $\frac{As}{Abar} = \frac{1409.8}{\frac{\pi}{4}16^2} = \frac{1409.8}{201} = 7.01 \approx 8$

6. Check if rebars can in be put in single layer

   b_{required} = 2 × cover+2 × stirrups diameter + No. of rebars × bar diameter + (No. of rebars-1) × spacing between rebars

   $b_{\text{required}} = 2*40+2*10+4*16+3*25 = 239$ mm

   $b_{\text{required}} = 239$ mm < b provided = 325 mm O.K

7. Compute the depth (h)

   h=550 mm

   Use (325 × 550) mm with 8φ16

8. Check the assumption of φ=0.9

   $a = \frac{As*fy}{0.85fc`*b} = \frac{1409.8*420}{0.85*25*325} = 85.74$ mm

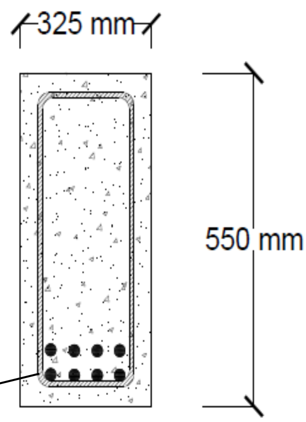
   $c = \frac{a}{\beta1} = \frac{85.74}{0.85} = 100.8$ mm

   $dt = 550-40-10-\frac{16}{2} = 492$ mm

   $\epsilon_t = \frac{dt-c}{c}\epsilon u = \frac{492-100.8}{100.8}*0.003 = 0.0116 > 0.005$

   $\therefore \varphi = 0.9$ ■

9. Draw final detailed section

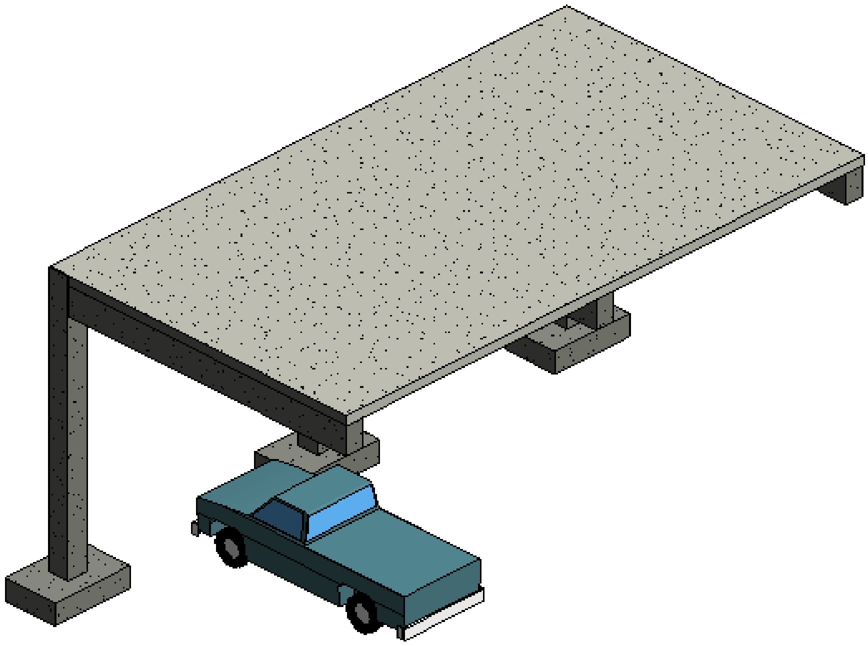


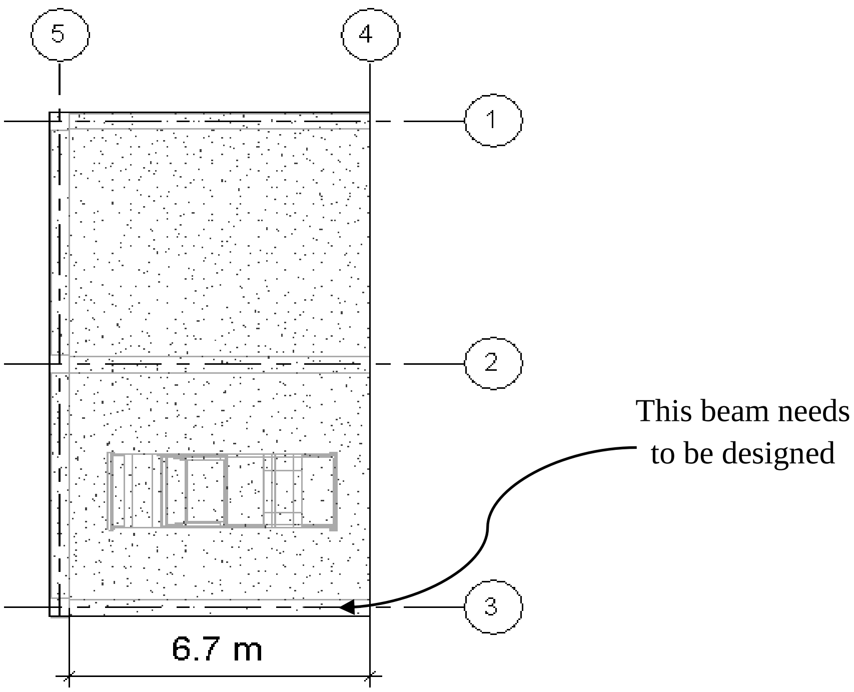
8φ16

Example 3: design a cantilever beam to support the slab shown in figure below, the beam is subjected to uniform factored load Wu=10 kN/m (including self-weight).

The designer intends to use:

- $\rho = 0.5\rho_{max}$ and $\frac{d}{b} = 2$.
- fc`=25 Mpa, fy=420 Mpa.
- Rebar 25mm for longitudinal reinforcement.
- Rebar 10mm for stirrups.



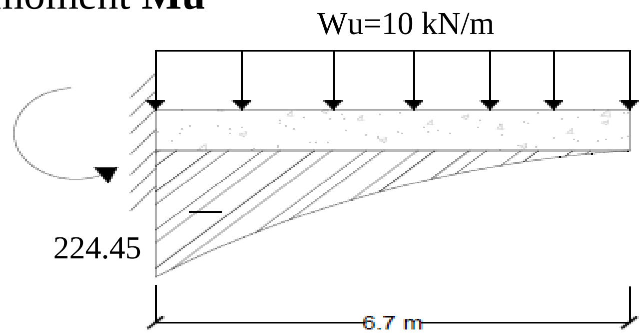


Solution:

1. Compute required factored applied moment **Mu**

   W_U=10 kN/m

   $M_u = \frac{Wu\ell^2}{2} = \frac{10*6.7^2}{2}$ =224.45 kN.m



2. $Mu=\varphi \mathbf{Mn}= \varphi\, \rho f_y \mathbf{bd^2(1\text{-}0.59}\frac{\rho f y}{f c`})$

   As it was mentioned in question $\rho=0.5\rho_{max}$ and $\frac{d}{b}$ =2.

   ρ_{max} =0.85*$\beta_1 \frac{fc`}{fy} \frac{\epsilon u}{\epsilon u+0.004}$ and ϵ_u =0.003

   ρ_{max} =0.85*0.85*$\frac{25}{420} \frac{0.003}{0.003+0.004}$ =0.0184

   $\rho=0.5\rho_{max}=0.5\times0.0184=9.2\times10^{-3}$

3. Substitute both of selected $\boldsymbol{\rho}$ and ration of $\frac{d}{b}$ in the main equation:

   $Mu=\varphi \mathbf{Mn}= \varphi\, \rho f_y \mathbf{bd^2(1\text{-}0.59}\frac{\rho f y}{f c`})$

   $224.45*10^6=0.9*9.2\times10^{-3}*420*b\ (b)^2*(1-0.59*\frac{9.2\times10^{-3}*420}{25})$

   $224.45*10^6 = 12.64*b^3$

   b=260 mm ≈275 mm

   So d= 2 *275=550 mm

4. Compute required steel area

   $As_{required} = \rho*(bd) = 9.2\times10^{-3} \times275\times550 = 1391.5\ mm^2$

5. Compute the required number of bars (n)

   No. of bars (n) = $\frac{As}{Abar} = \frac{1391.5}{\frac{\pi}{4}25^2} = \frac{1391.5}{491}$ =2.8 ≈ 3

6. Check if rebars can in be put in single layer

   $b_{required}$ = 2 × cover+2 × stirrups diameter + No. of rebars × bar diameter + (No. of rebars-1) × spacing between rebars

   $b_{required}$ = 2 *40+2*10+3*25+2*25 = 225 mm

   $b_{required}$=225 mm < b provided = 275 mm O.K

7. Compute the depth (h)

   $\mathbf{h_{for\ one\ layer}= d+cover+stirrups+ \frac{bar\ diameter}{2}}$

   h=550+40+10+12.5=612.5 mm ≈ 615 mm

Use (275 × 615) mm with 3φ25mm

8. Check the assumption of φ=0.9

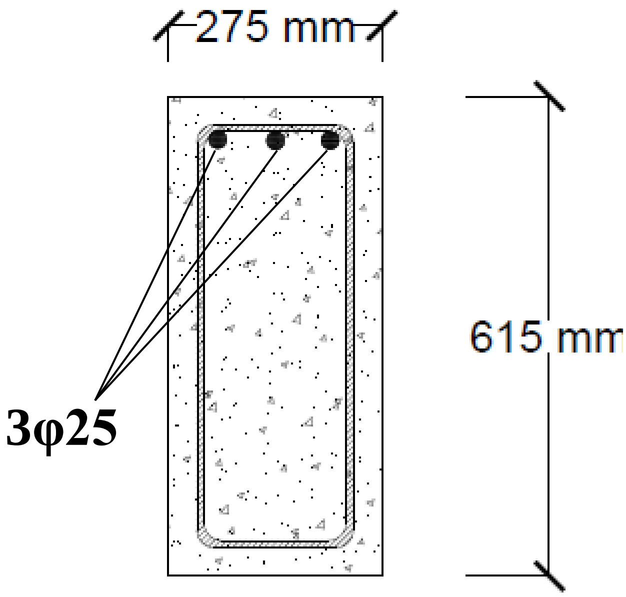
$$a = \frac{As*fy}{0.85fc`*b} = \frac{1391.5 \ *420}{0.85*25*275} = 100 \text{ mm}$$

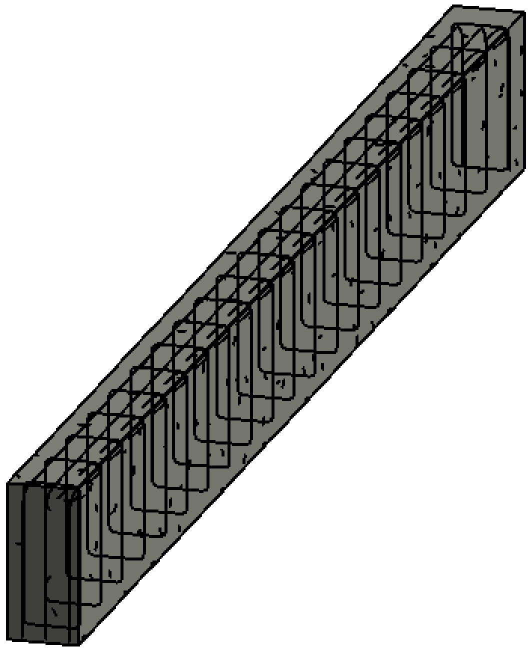
$$c = \frac{a}{\beta 1} = \frac{100}{0.85} = 117.64 \text{ mm}$$

$$\epsilon_t = \frac{dt-c}{c}\epsilon u = \frac{550-117.64}{117.64} *0.003 = 0.011 > 0.005$$

$\therefore \varphi$ =0.9

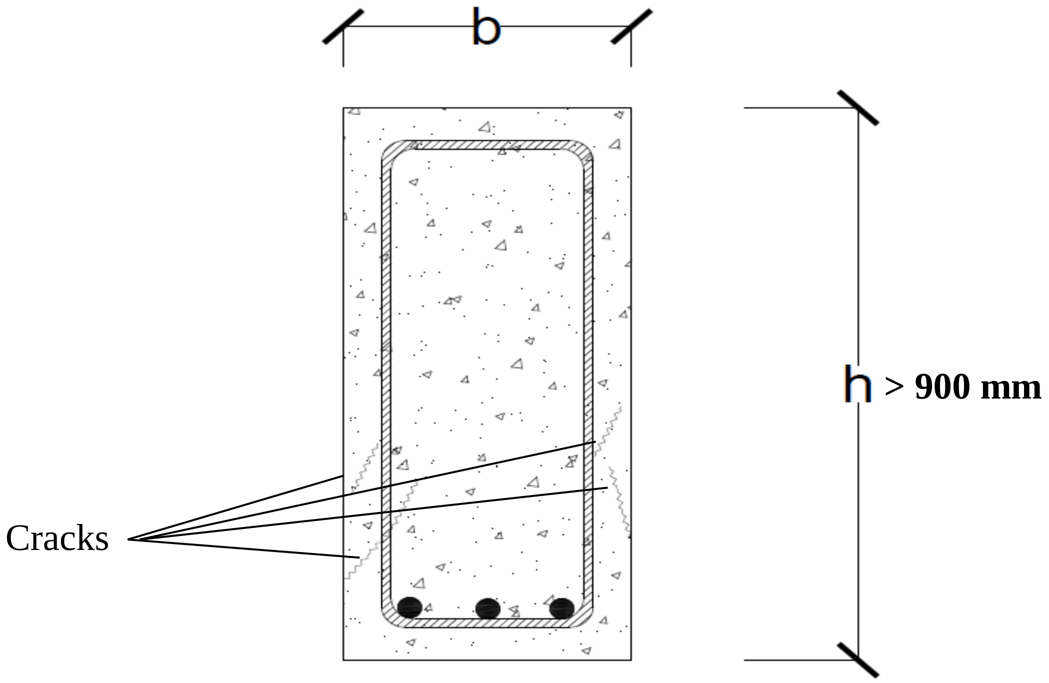
9. Draw final detailed section





Skin Reinforcement for beam ACI (9.7.2.3)

- For relatively deep beams, some reinforcement should be placed near the vertical faces of the tension zone to control cracking in the web may exceed the crack widths at the level of the flexural tension reinforcement.



- For beams have h exceeding 900 mm longitudinal skin reinforcement shall be uniformly distributed on both side faces of the beam for a distance h/2 from tension face.

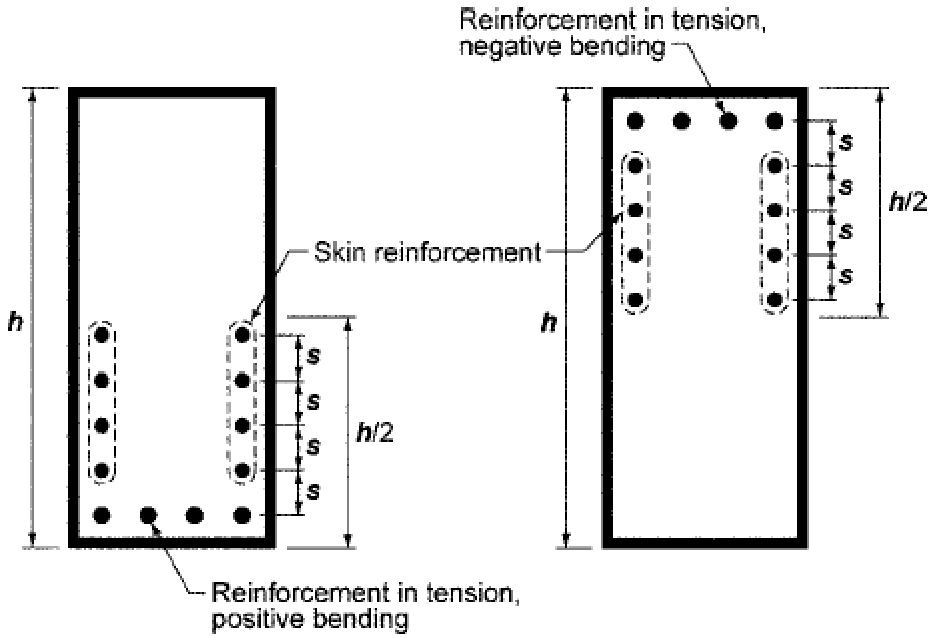


Fig. R9.7.2.3—Skin reinforcement for beams and joists with h > 900 mm.

- Spacing of skin reinforcement shall not exceed **s** given in **Table 24.3.2.**

Table 24.3.2—Maximum spacing of bonded reinforcement in nonprestressed and Class C prestressed one-way slabs and beams

Reinforcement type		Maximum spacing s
Deformed bars or wires	Lesser of:	$380\left(\frac{280}{f_s}\right) - 2.5c_c$
		$300\left(\frac{280}{f_s}\right)$
Bonded prestressed reinforcement	Lesser of:	$\left(\frac{2}{3}\right)\left[380\left(\frac{280}{\Delta f_{ps}}\right) - 2.5c_c\right]$
		$\left(\frac{2}{3}\right)\left[300\left(\frac{280}{\Delta f_{ps}}\right)\right]$
Combined deformed bars or wires and bonded prestressed reinforcement	Lesser of:	$\left(\frac{5}{6}\right)\left[380\left(\frac{280}{\Delta f_{ps}}\right) - 2.5c_c\right]$
		$\left(\frac{5}{6}\right)\left[300\left(\frac{280}{\Delta f_{ps}}\right)\right]$

S=lesser of $[380(\frac{280}{fs} - 2.5c_c),\ 300(\frac{280}{fs})]$

- S is center to center of reinforcement.
- Where c_c is the **clear cover** from skin reinforcement to **side face**.
- It will be permitted to take **fs = (2/3)* fy**
- The size of the skin reinforcement is not specified; research has indicated that the spacing rather than bar size is of primary importance. **Bar size 10mm** to **16mm** can be used.
- For the case of *fy*=420 Mpa and 50, clear cover to the primary reinforcement, with *fs*=280 Mpa , the maximum bar spacing is 250 mm.

Home Work: Design a simply supported rectangular reinforced concrete beam shown in Figure below.

Assume that the designer intends to use:

- Concrete of fc' =25 Mpa.
- Steel fy = 420 Mpa.
- Use $\rho = 0.5\rho max$ and h = 1000 mm
- Rebar diameter 25mm for longitudinal reinforcement.
- Rebar diameter 10mm for stirrups.
- W_D=10 kN/m**(including self-weight)**= and W_L=15 kN/m